

# Precursor Interceptor Guidance Using the Sliding Mode Approach

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Sliding mode guidance laws for intercepting high speed targets in a novel head-pursuit engagement are presented. The guidance laws impose a geometric relation that positions the interceptor missile ahead of the target, on its flight trajectory, so that both fly in the same direction. The missile speed is planned to be lower than that of the target, and therefore the target closes in on the interceptor missile. Using this approach the closing speed is significantly reduced relative to a head-on engagement; and compared to a tail-chase engagement, the low closing speed is achieved with reduced energy requirements. The guidance laws are derived for an aerodynamically controlled endo-atmospheric interceptor as well as for an exo-atmospheric kill vehicle controlled by divert thrusters. Simulation results confirm the viability of the proposed sliding mode guidance laws in several representative engagements against maneuvering high-speed targets.

## Nomenclature

$a$	normal acceleration
$e$	deviation from the geometric rule
$K$	speed ratio
$\mathcal{L}$	Lyapunov function
$m$	mass
$n$	guidance constant
$q$	discrete SMC design parameter
$r$	range
$u$	discrete controller
$V$	speed
$\Delta T$	pulse maximum duration
$\Delta$	modeling error
$\delta$	instantaneous interceptor direction of flight relative to the LOS.
$\epsilon$	discrete SMC design parameter
$\gamma$	flight path angle
$\lambda$	line of sight angle
$\mu$	gain
$\sigma$	sliding variable
$\tau$	time constant
$\theta$	instantaneous target direction of flight relative to the LOS.

### Subscripts

$c$	continuous
$d$	discrete
$eq$	equivalent
$I$	interceptor

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$r$	radial
$T$	target
$uc$	uncertainty
$\lambda$	perpendicular to LOS

#### *Superscripts*

$c$	command
$max$	maximum

#### *Abbreviations*

DPP	deviated pure pursuit
HP	head pursuit
KV	kill vehicle
LOS	line of sight
PN	proportional navigation
PP	pure pursuit
SMC	sliding mode control

## I. Introduction

INTERCEPTION of high speed targets such as ballistic missiles is a formidable challenge. Several anti-ballistic missile systems were developed in recent years for this task using intercepting missiles that are launched from the protected territory against the incoming threats. The interception in these scenarios is typically head-on, with very high closing speeds. This imposes severe requirements on the interceptor systems such as precise detection of the target from large distances by the onboard seekers, and very fast response time of the missile subsystems.

To overcome these difficulties, a different approach was suggested in Ref. 1, where the interceptor velocity is matched with that of the target by a preliminary maneuver. If the target path is predictable, as in the case of ballistic missiles, the maneuver is designed such that the interceptor missile is positioned ahead of the target on its predicted flight path, flying in the same direction but at a slightly lower speed. This way the target closes in on the interceptor that is conducting the necessary lateral maneuvers to achieve interception. The interceptor speed along the target future path can be selected to achieve a desired closing rate. A similar low closing speed can be obtained in a tail-chase scenario. However, tail-chase requires that the interceptor will be faster than the target and therefore more energy is needed during the preliminary maneuver to reach the desired chasing speed. This energy difference is significant in the ballistic missile defense scenario due to the high speed of the typical targets.

Various guidance methods have been examined for implementation in the different stages of exo or endo atmospheric interception scenarios of ballistic missiles. Some of these methods are described next. In Ref. 2 a modified version of proportional navigation (PN) guidance law<sup>3</sup> was proposed for implementation in the coast phase. A variable bias was applied to the actual line-of-sight (LOS) to account for engine burn. The terminal guidance in a hyper-velocity exo-atmospheric orbital interception was studied in Ref. 4. The control energy expenditure is reduced by constraining the expected final state to a function of the estimation error. An optimal guidance algorithm was proposed in Ref. 5 for the interception of a non-maneuvering target decelerated by atmospheric drag. Its implementation requires knowledge on many scenario states, obtained from a nonlinear state estimator. In a recent paper<sup>6</sup> a differential game guidance law was proposed against targets having known speed and lateral acceleration limit profiles. It was shown that in a ballistic missile interception scenario such a guidance law provides a significant improvement in the homing accuracy compared to a guidance law derived based on a model with constant velocities and lateral acceleration limits. Although all of the above mentioned guidance laws may provide interception in some scenarios, the actual geometry of the engagement is not imposed. It is a result of the specific initial conditions, target maneuvers, and guidance law used.

The simple guidance law of pure pursuit (PP) and its derivative deviated PP (DPP)<sup>7</sup> can impose a final interception geometry. In PP the interceptor is aimed at the target and it is intercepted from its rear.<sup>8</sup> The DPP guidance law is an extension of PP in that the missile is aimed at a constant lead angle to the target and the target is intercepted from a constant angle, dependent on the lead angle and speed ratio.<sup>7</sup> Such guidance laws require minimal knowledge of the interception state variables and future target

maneuvers. However, they impose severe maneuver requirements from the interceptor (especially PP) and require a velocity advantage of the interceptor. Imposing an impact angle can also be performed by a circular navigation guidance law.<sup>9</sup> The name of this guidance law comes from its basic principle of following an arc of a circle to the target. A more complicated guidance law that can impose initial and final flight path angles was introduced in Ref. 10. The algorithm requires the solution of a two point boundary value problem and thus can be implemented against targets with known trajectories.

Imposing the impact angle alone will not enable following the unique geometry outlined in Ref. 1. It is also needed that the target's head will be hit while the interceptor has a speed disadvantage. In a recent paper by the authors<sup>11</sup> it was shown that by maintaining a simple geometric rule, the trajectory proposed in Ref. 1 can be achieved. The term head pursuit (HP) was coined to indicate that interception is aimed at always hitting the target's head. In essence the proposed geometric rule is equivalent of being in sliding mode.

The sliding mode control (SMC) methodology is a well known method described in many papers and textbooks.<sup>12–14</sup> It is an intuitive and simple robust control technique, addressing highly nonlinear systems with large modeling errors and uncertainties. The SMC methodology has been successfully used in various guidance applications. A missile guidance law in the class of PN, derived using the SMC approach, was proposed in Ref. 15. The sliding surface was selected to be proportional to the LOS rate and the target maneuvers were considered as bounded uncertainties. Using numerical simulations, the superiority of the proposed guidance law over the conventional PN was advocated. In Ref. 16 an adaptive sliding mode guidance law was derived. Using analysis and simulations, robustness to disturbances and parameter perturbations was shown.

In this paper we develop HP sliding mode guidance laws for an aerodynamically controlled endo-atmospheric interceptor as well as for an exo-atmospheric kill vehicle controlled by divert thrusters. The continuous and discrete time controllers enable maintaining the geometric relation that leads to the particular HP engagement. In the next section, the HP interception engagement is outlined and the required geometric relations are described. Then, sliding mode controllers are developed both for the continuous and discrete cases. Following, the performance of the proposed SMC HP guidance laws is examined through simulation and the results show the viability of the proposed designs. Concluding remarks are offered in the last section.

## II. Head Pursuit Interception

The schematic view of the proposed engagement outline is shown in Fig. 1. The interceptor trajectory has three stages: first stage for approach, second for trajectory bending and third, endgame, in which the kill vehicle (KV) conducts final corrective maneuvers. The missile is launched towards the ballistic target in a head-on trajectory, and at a predetermined time is steered to bend its flight trajectory until reaching a so-called trajectory matching flight mode. The drawing illustrates exo-atmospheric interception in which thrust is used to achieve trajectory bending and end-game maneuvers.

In the beginning of the endgame phase the interceptor is flying close to the target predicted trajectory, ahead of the target but at a lower speed. The objective of the endgame guidance is to correct the position and flight direction errors. Note the unconventional final geometry in which the target approaches the interceptor from its rear end. Using this approach the closing speed is significantly reduced relative to a head-on engagement; and compared to a tail-chase engagement, the low closing speed is achieved with reduced energy requirements. Such geometry also relaxes the requirements from the interceptor seeker dome since it is not exposed to the high aerodynamic heating. However, it requires special adaptation of the kill mechanism.

A roll controlled interceptor is considered in this paper. For the relatively short time interval of the endgame (with small changes in the flight direction) the motion of such an interceptor can be separated into two perpendicular channels and the guidance problem can be treated as planar in each of those channels. The planar endgame geometry is shown in Fig. 2. The target  $T$  is located behind the slower interceptor  $I$ . The speed, maneuvering acceleration, and flight path angle are denoted by  $V$ ,  $a$ , and  $\gamma$ , respectively; the range between the target and interceptor is  $r$ , and  $\lambda$  is the LOS angle relative to a fixed reference. The angles  $\theta$  and  $\delta$  are the instantaneous target and interceptor direction of flight relative to the LOS.

The engagement kinematics is expressed in a polar coordinate system  $(r, \lambda)$  attached to the target

$$\dot{r} = V_r \tag{1a}$$

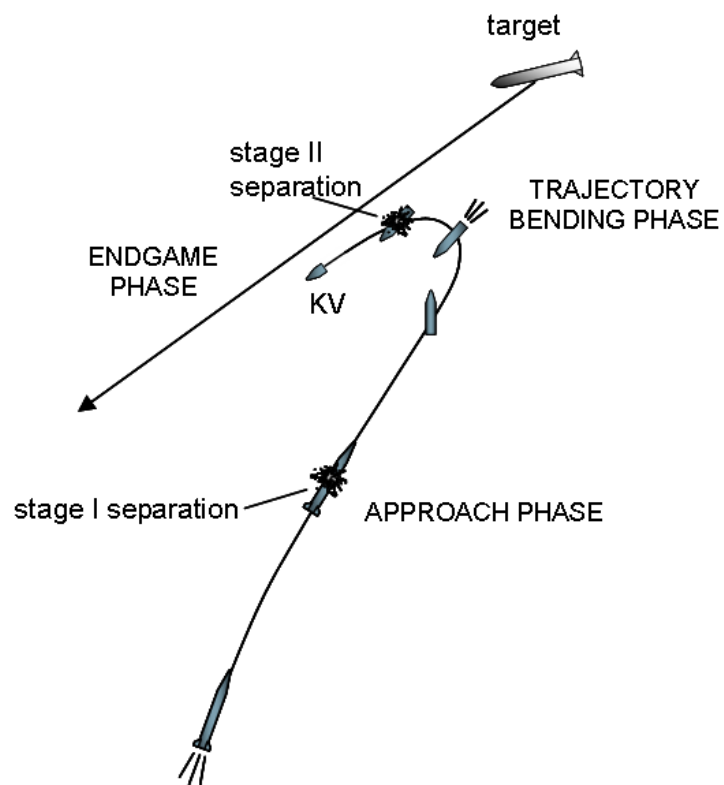


Figure 1. Head pursuit scenario.

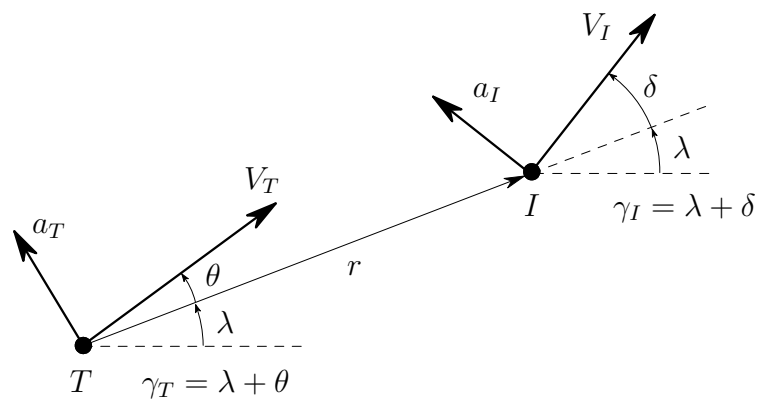


Figure 2. Planar engagement geometry.

$$\dot{\lambda} = V_{\lambda}/r \quad (1b)$$

where the closing speed  $V_r$  is

$$V_r = V_I \cos \delta - V_T \cos \theta \quad (2)$$

and the speed perpendicular to the line of sight is

$$V_{\lambda} = V_I \sin \delta - V_T \sin \theta \quad (3)$$

We assume that the interceptor and target speeds,  $V_I$  and  $V_T$ , are constant and define the non-dimensional parameter  $K$  as the speed ratio

$$K = V_I/V_T < 1 \quad (4)$$

Note that we analyze a HP interception scenario in which the interceptor is designed with a speed disadvantage.

The lateral accelerations  $a_I$  and  $a_T$  determine the interceptor and target trajectories

$$\dot{\gamma}_I = a_I/V_I \quad (5a)$$

$$\dot{\gamma}_T = a_T/V_T \quad (5b)$$

where the flight path angles  $\gamma_I$  and  $\gamma_T$  satisfy

$$\gamma_I = \lambda + \delta \quad (6a)$$

$$\gamma_T = \lambda + \theta \quad (6b)$$

Using Eqs. (5),(6) we obtain

$$\dot{\delta} = a_I/V_I - V_{\lambda}/r \quad (7a)$$

$$\dot{\theta} = a_T/V_T - V_{\lambda}/r \quad (7b)$$

It is required that near interception ( $r \rightarrow 0$ ) both interceptor and target will fly in the same direction, along the LOS. Thus, the objective of the guidance law is to impose

$$\theta(r \rightarrow 0) \approx 0 \quad (8)$$

and

$$\delta(r \rightarrow 0) \approx 0 \quad (9)$$

By maintaining the interception lead angle proportional to the target flight direction relative to the LOS the unique HP geometry can be attained. This geometric rule is

$$\delta = n\theta \quad (10)$$

where  $n$  is the guidance constant.

The conditions for perfect intercept given an ideal HP scenario with no target maneuvers and interceptor maneuver dynamics and bounds is given in the next two theorems.

**Theorem 1.** *Against a non-maneuvering target, a necessary condition for performing the HP interception is*

$$n > 1/K \quad (11)$$

**Proof:** see Ref. 11. □

**Theorem 2.** *Against a non-maneuvering target, and given  $n > 1/K$ , the sufficient condition for performing the HP interception is*

$$|\theta| < \sqrt{6 \frac{Kn - 1}{Kn^3 - 1}} \quad (12)$$

**Proof:** see Ref. 11. □

### III. Continuous Time Head Pursuit Guidance

In this section we analyze an endo-atmospheric HP interception scenario with modeling errors and target maneuvers. We assume that both adversaries have aerodynamic control surfaces and thus the interception problem can be analyzed in continuous time. The derivation is performed using the sliding mode approach in continuous time.

#### A. Background

Most of the early works on sliding mode control was with respect to continuous time systems.<sup>17</sup> The controller is obtained by converting an  $n$ -th order tracking problem to a first order stabilization problem. The design is performed around a sliding surface commonly denoted by  $\sigma = 0$ , where the sliding variable  $\sigma$  is a function of the system tracking error and possibly its derivatives. The problem is to drive the scalar quantity defining the sliding surface to zero, and maintain it there, ultimately achieving exact tracking. When the system response is confined to the sliding surface, it is said that the system is in a sliding mode.

Next, we present the adversaries dynamics, define the continuous sliding variable and derive the continuous time sliding mode HP controller.

#### B. Dynamics

We assume that both the interceptor and target closed loop dynamics can be represented by equivalent first order transfer functions

$$\dot{a}_I = (a_I^c - a_I) / \tau_I + \Delta_I \quad (13)$$

$$\dot{a}_T = (a_T^c - a_T) / \tau_T + \Delta_T \quad (14)$$

where  $\tau_I$  and  $\tau_T$  are the respective interceptor and target time constants; and  $\Delta_I$ ,  $\Delta_T$  are their bounded modeling errors.

$$|\Delta_I| \leq \bar{\Delta}_I \quad (15)$$

$$|\Delta_T| \leq \bar{\Delta}_T \quad (16)$$

We also assume that the target's acceleration command is bounded

$$|a_T^c| \leq a_T^{max} \quad (17)$$

#### C. Sliding Variable

We define the deviation from the geometric rule of Eq. (10) as

$$e = \delta - n\theta \quad (18)$$

Since the relative degree of this error with respect to the control command is two, the sliding variable should include a first derivative of the error  $e$ . This will guarantee that the control will have direct effect on the sliding surface. Thus, the SMC sliding variable, for the continuous case, is defined as

$$\sigma_c = e + \tau \dot{e} \quad (19)$$

The time constant  $\tau$  can be selected such that the error  $e$  diminishes to zero at the required rate, regardless of the value of the control. Substituting Eqs. (5), (6), and (18) in Eq. (19) we can write the sliding variable as

$$\sigma_c = \gamma_I - n\gamma_T + (n-1)\lambda + \tau [a_I/V_I - na_T/V_T + (n-1)V_\lambda/r] \quad (20)$$

The dynamics of this sliding variable is

$$\begin{aligned} \dot{\sigma}_c = & a_I/V_I - na_T/V_T + (n-1)V_\lambda/r \\ & + \tau [(a_I^c - a_I)/\tau_I/V_I + \Delta_I/V_I - n(a_T^c - a_T)/\tau_T/V_T - n\Delta_T/V_T] \\ & + \tau(n-1)(\dot{V}_\lambda r - V_\lambda V_r)/r^2 \end{aligned} \quad (21)$$

where  $V_r$  and  $V_\lambda$  are given in Eqs. (2),(3) and

$$\dot{V}_\lambda = a_I \cos \delta - a_T \cos \theta - V_\lambda V_r/r \quad (22)$$

## D. Sliding Mode Controller

The sliding mode controller  $a_I^c$  consists of an equivalent part denoted  $a_{Ieq}^c$  and an uncertainly part denoted  $a_{Iuc}^c$ , where

$$a_I^c = a_{Ieq}^c + a_{Iuc}^c \quad (23)$$

The equivalent controller is designed to maintain the system on the sliding surface, by imposing  $\dot{\sigma} = 0$  in the absence of modeling errors and target maneuvers. Thus, from Eq. (21) with  $\Delta_{(\cdot)} = 0$  and  $a_T^c = 0$  we obtain

$$a_{Ieq}^c = f_I a_I + f_T a_T + f_\lambda \dot{\lambda} \quad (24)$$

where

$$f_I = 1 - \tau_I/\tau - \tau_I(n-1)V_I \cos \delta/r \quad (25)$$

$$f_T = Kn(\tau_I/\tau - 1) - V_I(n-1)\tau_I \cos \theta/r \quad (26)$$

$$f_\lambda = V_I \tau_I(n-1)(2V_r/r - 1/\tau) \quad (27)$$

Modeling errors and target maneuvers will cause the system to depart from the sliding surface. The uncertainty controller is designed to drive the system to the sliding surface in finite time in the face of these errors and uncertainties. Here, the design of the uncertainty controller is based on the model of the interceptor and target dynamics and the bounds given in Eqs. (13)-(17). We select the uncertainty controller in the form

$$a_{Iuc}^c = \tau_I V_I \mu_c \text{sign}(\sigma_c) \quad (28)$$

The gain  $\mu_c$  is obtained from analyzing the following candidate Lyapunov function

$$\mathcal{L}_c = \frac{1}{2} \sigma_c^2 \quad (29)$$

Using the time derivative of this candidate Lyapunov function, the well-known reaching condition is obtained

$$\dot{\mathcal{L}}_c = \sigma_c \dot{\sigma}_c < 0 \quad (30)$$

This derivative with the equivalent and uncertainty controllers of Eqs. (24), (28), is

$$\dot{\mathcal{L}}_c = \sigma_c \dot{\sigma}_c = \tau \sigma_c [\mu_c \text{sign}(\sigma_c) + \Delta_I/V_I - n\Delta_T/V_T - na_T^c] \quad (31)$$

Using the bounds from Eqs. (15)-(17) this derivative can be bounded by

$$\dot{\mathcal{L}}_c \leq \tau |\sigma_c| [\mu_c + \bar{\Delta}_I/V_I + n\bar{\Delta}_T/V_T + na_T^{max}] \quad (32)$$

To obtain finite time convergence to the sliding surface we choose

$$\mu_c < -[\bar{\Delta}_I/V_I + n\bar{\Delta}_T/V_T + na_T^{max}] \quad (33)$$

such that negative definiteness of the Lyapunov function is ensured. Note that the gain  $\mu_c$  is proportional to  $n$ . To obtain the largest interception envelope,  $n$  should be chosen based on the results of *Ref. 11*. Nonetheless, it should not be chosen too large so that the interceptor's maneuver capability will not be exceeded.

## IV. Discrete Time Head Pursuit Guidance

In space applications, control is achieved by divert thrusters that are typically on-off with constant level of thrust. For this type of application, we derive a discrete sliding mode controller.

### A. Background

Techniques to design discrete time sliding mode controllers were proposed in *Ref. 18* and *19*. Since in discrete time exact tracking of the sliding surface can not be obtained the notion of quasi-sliding mode was introduced.<sup>20</sup> It has three attributes:<sup>21</sup> A1) The trajectory, starting from the initial conditions, monotonically approaches the switching surface and crosses it in finite time, A2) In successive sampling periods the trajectory crosses the switching surface, resulting in a zigzag motion around the switching surface, A3) The distance from the switching surface is non-increasing and the trajectory stays within a specified band. Conditions to ensure discrete sliding mode were derived in *Ref. 20* and *22*.

Next, we present the adversaries dynamics. Then, we define the discrete sliding variable. This is followed by the derivation of the discrete time sliding mode HP controller.

## B. Dynamics

We assume that in an exo-atmospheric interception scenario the interceptor and target maneuvers are with instantaneous, ideal, dynamics such that

$$a_I = a_I^c \quad (34a)$$

$$a_T = a_T^c \quad (34b)$$

where  $a_T^c$  is the bounded target's acceleration (see Eq. (17)). We assume that the acceleration of the interceptor is obtained using on-off divert thrusters with a constant thrust level  $T_0$ . Thus, the level of the missile on-off acceleration is

$$a_I^{max} = \frac{T_0}{m} \quad (35)$$

where  $m$  is the interceptor's current mass. The missile controller is  $u$  representing the amount of time and direction the thrusters will be open in the next interval  $\Delta T$ . Thus,

$$|u| \leq \Delta T \quad (36)$$

## C. Sliding Variable

The relative degree of the error (Eq. (18)) to the interceptor's control command is one. Thus, we define the discrete sliding variable as

$$\sigma_d = e = \delta - n\theta \quad (37)$$

Considering the discrete nature of the problem and substituting Eqs. (6) we can write

$$\sigma_d(k) = \gamma_I(k) - n\gamma_T(k) + (n-1)\lambda(k) \quad (38)$$

## D. Sliding Mode Controller

The discrete equivalent of the reaching condition from Eq. (30) is

$$[\sigma_d(k+1) - \sigma_d(k)]\sigma_d < 0 \quad (39)$$

However, as pointed out in Ref. 20, Eq. (39) is a necessary condition for discrete sliding mode but it is not sufficient, as the trajectory may zigzag around the sliding surface with increasing magnitude. Thus, in Ref. 22 the following reaching condition was suggested

$$|\sigma_d(k+1)| < |\sigma_d(k)| \quad (40)$$

which is equivalent to the condition<sup>19</sup>

$$\mathcal{L}_d(k+1) < \mathcal{L}_d(k) \quad (41)$$

where  $\mathcal{L}_d$  is the discrete equivalent of the continuous time candidate Lyapunov function from Eq. (29).

To meet the conditions from Eqs. (40),(41) Gao<sup>21</sup> suggested the following reaching law

$$\sigma_d(k+1) - \sigma_d(k) = -q\Delta T\sigma_d(k) - \epsilon\Delta T\text{sign}[\sigma_d(k)] \quad (42)$$

where  $\epsilon, q$  are design parameters selected such that  $\epsilon > 0$ ,  $q > 0$ , and  $1 - q\Delta T > 0$ .

Using Eqs. (5), (7), (38) we obtain the following expression for  $\sigma_d(k+1)$

$$\sigma_d(k+1) = \sigma_d(k) + \frac{T_0/m}{V_I}u - \Delta a_T + \frac{(n-1)V_\lambda(k)}{r(k)}\Delta T + \Delta e \quad (43)$$

where

$$|\Delta a_T| \leq \bar{\Delta}a_T \quad ; \quad \bar{\Delta}a_T = na_T^{max}\Delta T/V_T \quad (44)$$

and  $\Delta e$  is the bounded series expansion error

$$|\Delta e| \leq \bar{\Delta}e \quad (45)$$

Substituting Eq. (43) in (42) we obtain the required duration and direction the on-off divert thrusters must be operated

$$u^{req} = \frac{V_I}{T_0/m} \left[ \Delta a_T - \frac{(n-1)V_\lambda(k)}{r(k)} \Delta T - \Delta e - q \Delta T \sigma_d(k) - \epsilon \Delta T \text{sign}(\sigma_d(k)) \right] \quad (46)$$

Since  $\Delta a_T$  and  $\Delta e$  are uncertainties only their bounds are known a priori. Also, the thrusters operation duration is bounded by  $\Delta T$ . Thus, we choose the following controller

$$u = \Delta T \text{sat} \left[ \frac{V_I}{T_0/m} \left( -\frac{(n-1)V_\lambda(k)}{r(k)} - q \sigma_d(k) - (\epsilon + \bar{\Delta} a_T / \Delta T + \bar{\Delta} e / \Delta T) \text{sign}(\sigma_d(k)) \right) \right] \quad (47)$$

where  $\text{sat}$  is the standard saturation function.

Note that only if there are no uncertainties (i.e.  $\Delta a_T = 0$  and  $\Delta e = 0$ ) and if the control does not saturate then it can be ensured that the system can remain in quasi sliding mode (i.e. within a non increasing band around the sliding surface). Note that if  $\Delta T \rightarrow 0$  then it is ensured that the system will remain in ideal sliding mode after the sliding surface is reached.

## V. Performance Analysis

In this section a scenario with a maneuvering target is studied using simulations. Two different interceptors are examined: one with a continuous controller and the other employing a bang-bang maneuver device. In both cases perfect information is assumed.

The simulation parameters for the endo-atmospheric interception scenario are summarized in Table 1.

Missile	Target	Kinematics	Guidance
$V_I = 1600m/s$	$V_T = 1900m/s$	$r_0 = 3km$	$n = 2$
$\tau_I = 0.2sec$	$a_T = [-20, 0, 20]g$	$\theta_0 = -20 \text{ deg}$	$\tau = 0.2sec$
	$\tau_T = 0.2sec$	$\delta_0 = -20 \text{ deg}$	

Table 1. Simulation parameters for continuous case.

Three different scenarios are investigated: a non-maneuvering target, and targets performing constant maneuvers of  $20g$  or  $-20g$ . Fig. 3 presents the interceptor and target trajectories in an inertial coordinate system. Despite the large initial heading error of 20 degrees, the interceptor gradually approaches the target future trajectory and tracks it until it is caught up by the target.

In Fig. 4 the interceptor relative trajectories in target fixed coordinates are shown. It can be seen that even in the presence of target maneuvers and initial heading errors interception is achieved with small  $\theta$  as required.

The missile acceleration profiles are plotted in Fig. 5. It can be observed that the acceleration approaches that of the target as the two vehicles come nearer. In the case of a maneuvering target, the acceleration difference at interception is due to the different speeds and turning radius of the two vehicles. The difference in the scenario duration, due to the different interception trajectories, is also evident.

Finally, the value of the sliding function is shown in Fig. 6. After a short transient,  $\sigma$  is kept close to zero up to the intercept point.

Next, the bang-bang controller of section IV.D is applied. We examine the same scenario for a non maneuvering target as in the continuous case. The simulation parameters are summarized in Table 2.

Fig. 7 presents the interceptor and target trajectories in an inertial coordinate system. As for the case with continuous unbounded controller, despite the large initial heading error of 20 degrees relative to the direction required by the guidance law, the interceptor performs the HP interception. The radius of turn for the bang-bang controlled interceptor is larger due to the saturated acceleration. Yet, as shown in Fig. 8, interception is achieved with a very small heading error. The error in  $\theta$ , due to the dead zone in the control logic and the time delay, represents the design trade-offs between system cost and performance.

The acceleration profiles are plotted in Fig. 9. The bang-bang nature of the maneuvers is immediately evident. The maneuver starts with initial thrusting in the negative direction followed by positive thrusting

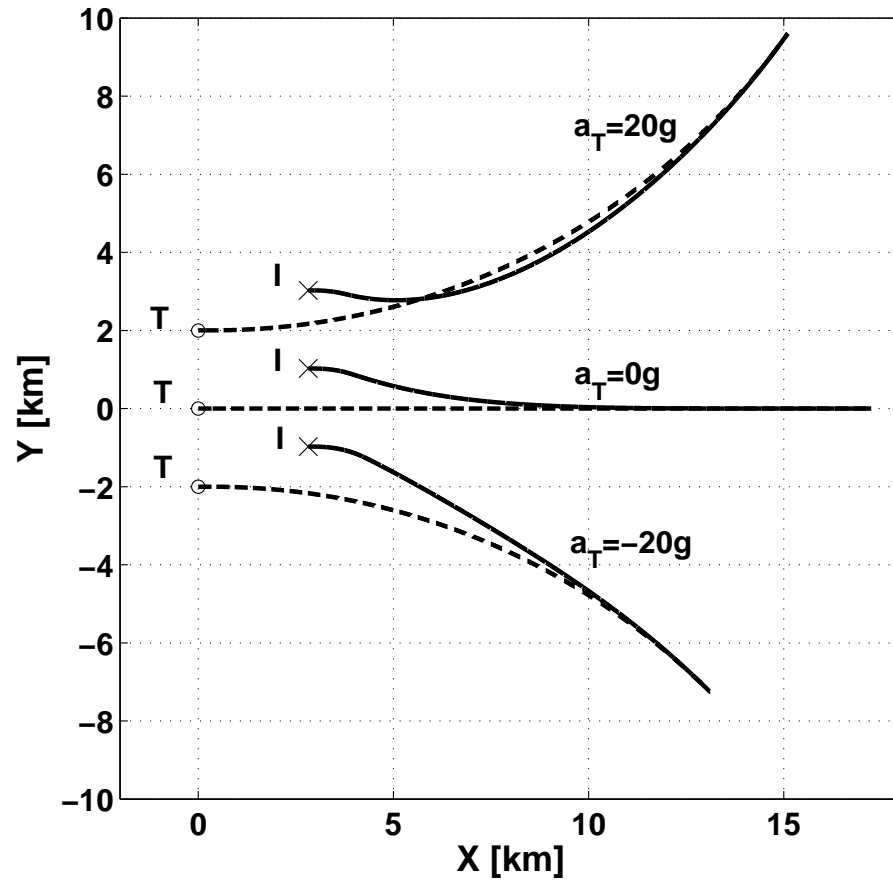


Figure 3. Inertial trajectories; continuous interceptor controller.

Missile	Target	Kinematics	Guidance
$V_I = 1600m/s$	$V_T = 1900m/s$	$r_0 = 3km$	$n = 2$
$T = 4000N$	$a_T = 0g$	$\theta_0 = -20 \text{ deg}$	$q = 0.5$
$m_0 = 40kg$	$\Delta a_T = 3g$	$\delta_0 = -20 \text{ deg}$	$\epsilon = 0.05$
$I_{sp} = 250sec$			

Table 2. Simulation parameters for discrete case.

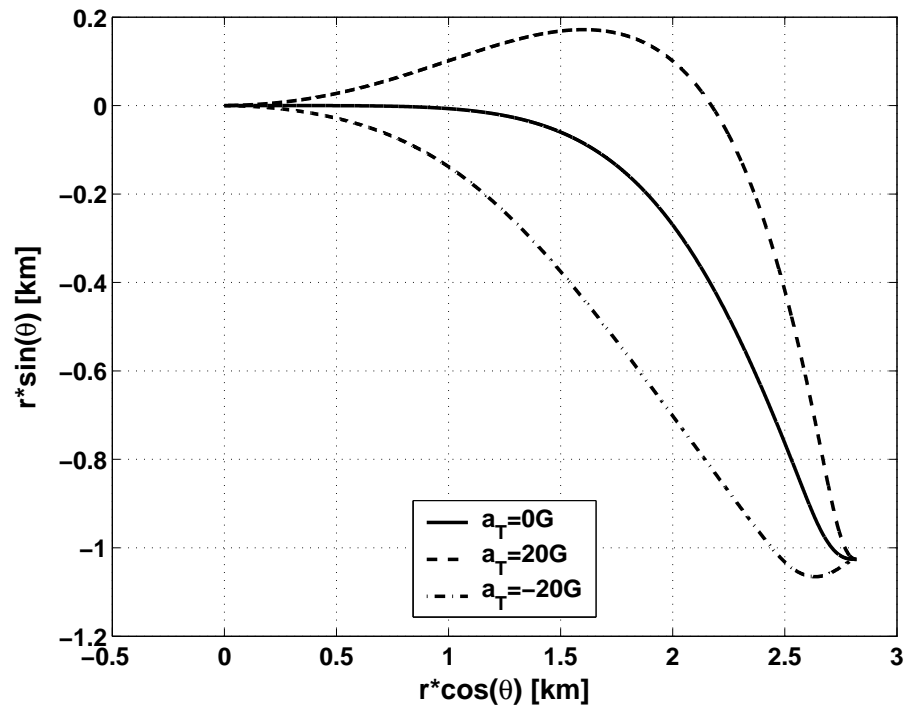


Figure 4. Relative trajectories in target fixed coordinates; continuous interceptor controller.

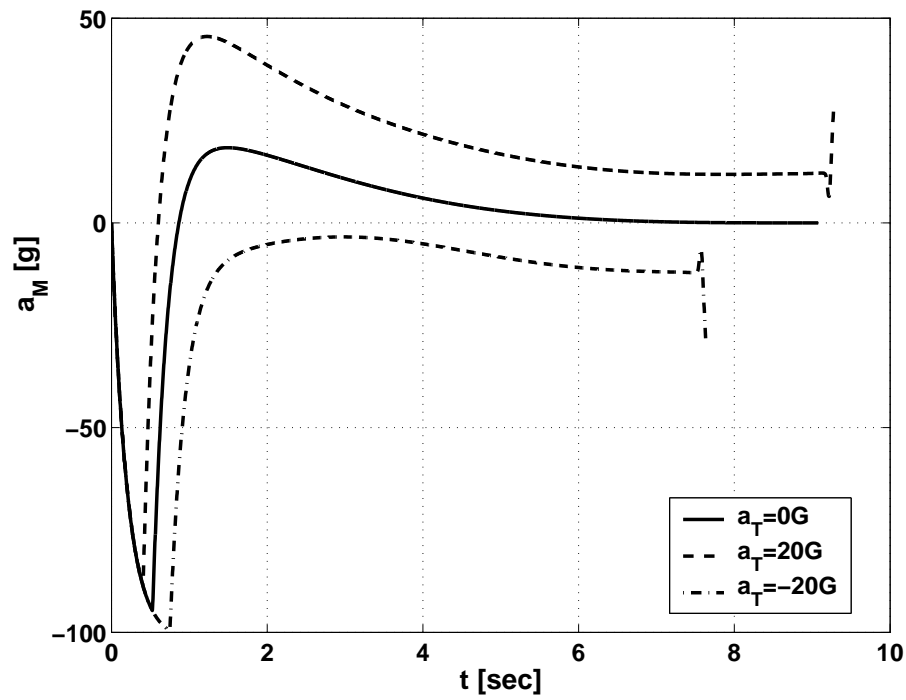


Figure 5. Missile acceleration profile; continuous unbounded interceptor controller.

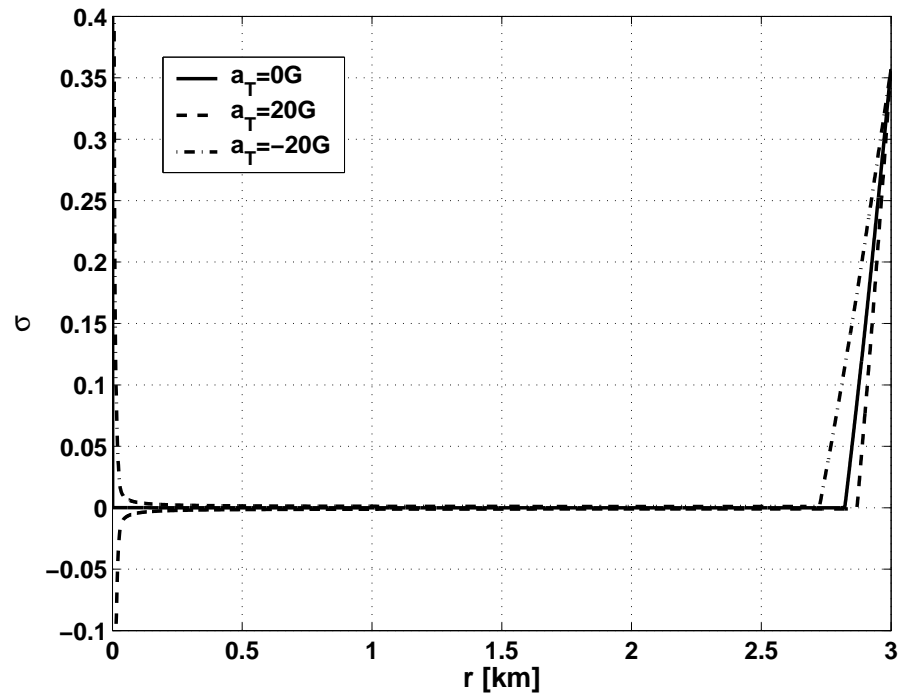


Figure 6. Sliding function value; continuous unbounded interceptor controller.

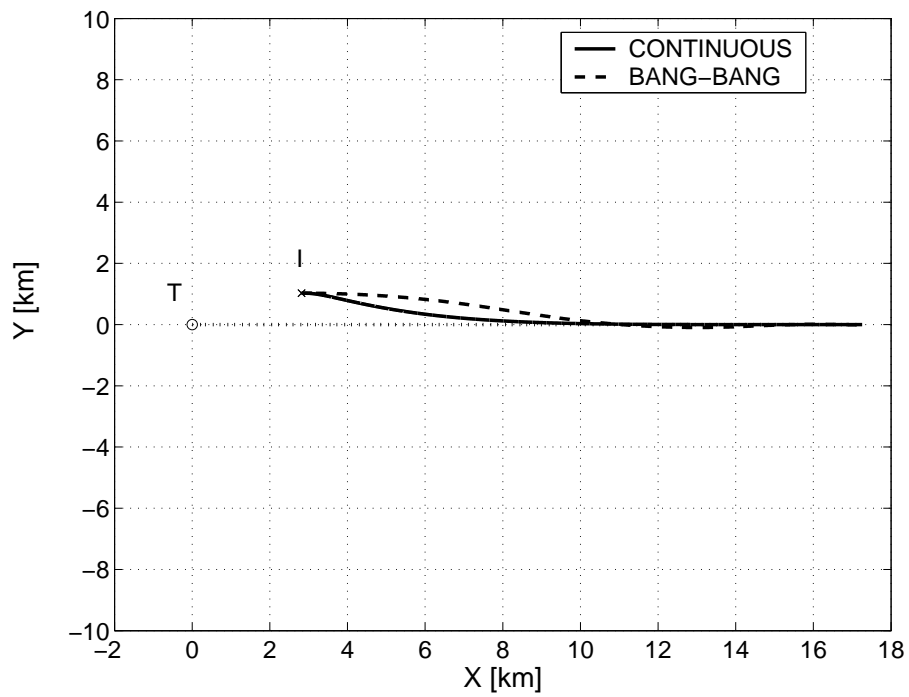


Figure 7. Inertial Trajectories; continuous and bang-bang interceptor controllers.

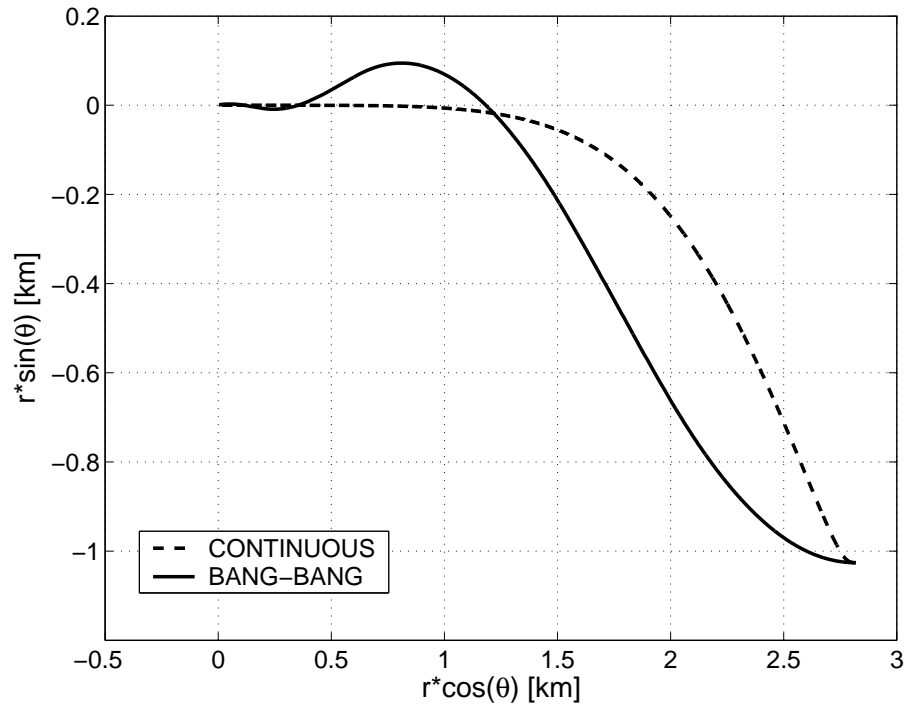


Figure 8. Relative trajectories in target fixed coordinates; continuous and bang-bang interceptor controllers.

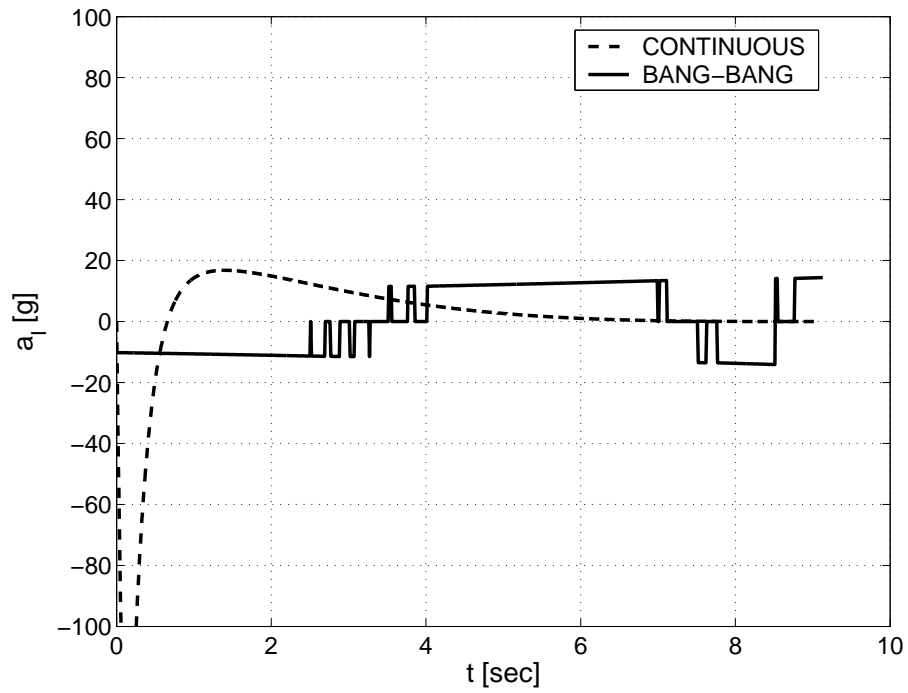


Figure 9. Interceptor acceleration profile; continuous and bang-bang controllers.

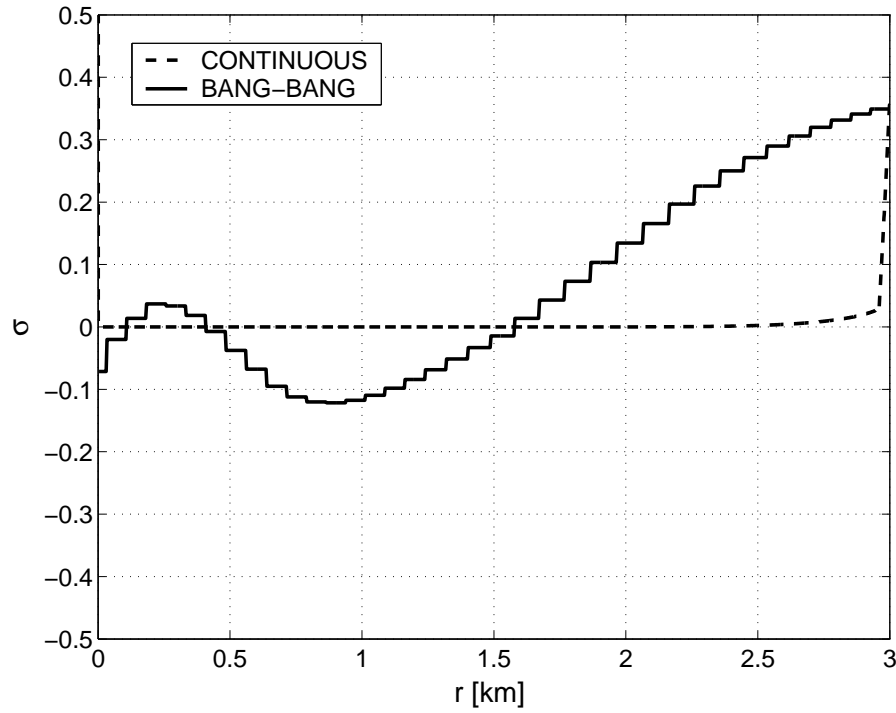


Figure 10. Sliding function; continuous and bang-bang interceptor controllers.

in order to decelerate and bring the interceptor to a halt in the lateral direction. When the desired flight direction is reached, the controller maintains the flight course by short corrections, each time the directional error exceeds the imposed limit. Note the increasing level of acceleration due to the propellant mass expenditure. Finally, the sliding functions are shown in Fig. 10. In contrast to the continuous case, it is not possible to bring the system to stay on the sliding surface. Rather, the controller keeps the interceptor close enough to the surface to achieve the desired interception.

## VI. Conclusions

Sliding mode guidance laws have been derived for both endo and exo atmospheric interceptors. It was shown that the proposed guidance laws enable positioning an interceptor in front of a high speed target and intercepting it in a novel head-pursuit geometry. Although the interceptor is planned to have a lower speed than the target, interception can be achieved even if the target maneuvers and there are large initial heading errors.

For the endo-atmospheric continuous implementation, the sliding mode guidance law allows bringing the system to the chosen sliding surface, imposing the required geometric relations, and the system remains on it as long as there are no target maneuvers and modeling errors. In the exo-atmospheric discrete, bang-bang, implementation the system can not remain on the chosen sliding surface but is contained within a boundary layer around it. Based on the required homing accuracy the width of the boundary layer can be chosen, imposing restrictions on the bang-bang amplitude and frequency.

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